DETERMINATION OF WALL SHEAR STRESS FROM VELOCITY PROFILE MEASUREMENT IN THE OUTER PART OF THE BOUNDARY LAYER

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The shear stress on a wall is determined through a comparison of experimental and theoretical determinations of velocity distribution using a family of Thompson or Cole profiles.

The shear stress on a wall is one of the most important characterisitcs of flow in a boundary layer. This characteristic is used in constructing various types of generalizations and in integral methods of calculating drag.

Clauser's method [1] is the simplest of the methods presently available for empirically determining local shear stress on a wall. It does not require special measuring instruments, such as are needed in determinations of shear stress made by heat sensors, floating elements, Preston surface tubes, etc. The method is based on the following law for the wall

$$\frac{u}{u_1} = \omega\left(\frac{1}{k}\ln\frac{y\omega u_1}{\nu} + B\right),\ \omega = \sqrt{\frac{C_f}{2}}. \quad (1)$$

By arranging the boundary-layer velocity distribution measured with a head meter or hot-wire anemometer in accordance with Eq. (1), we can find the value of C_F. Such a problem is usually solved graphically, by constructing a grid of straight lines $u/u_1 = f(yu_1/\nu)$ with the parameter ω.

We will use the method with a gradient flow and a flow with a high degree of turbulence in its outer portion. This can be done thanks to the universality of the wall law [2-4]. However, the section of the velocity profile which coincides with the logarithmic law is abbreviated when high positive pressure gradients are present, thus lowering the accuracy of the shear stress determination. In connection with this, Pirs and Tsimmerman [5] developed a universal method which makes it possible to use additional points of the velocity profile in the laminar and transitional regions of the boundary layer when analyzing experimental data. The number of experimental points of the velocity profile used to determine the averaged value of C_f is thus also increased. However, the computing procedures here are complicated appreciably. The reduction in the accuracy of velocity measurement in the immediate vicinity of the wall should also be considered.

In the generalization of Clauser's method proposed here, the region of empirical velocity values by which shear stress is determined is expanded at the expense of the outer part of the boundary layer. The velocity measurements are most accurate in this region.

To determine the wall shear stress in an assigned section x, we use the empirical values of velocity distribution u/u_1 and displacement thickness δ^*. Similar to Clauser's method, the distribution $u/u_1 = f(Re_y)$ at different values of C_f is compared with the empirical velocity profile. Since the velocity profile in the outer part of the boundary layer depends on the pressure gradient, then instead of the wall law we use the more universal approximation of velocity distribution after Thompson [6]

$$\frac{u}{u_1} = \gamma\left(\frac{u}{u_1}\right)_{in} + (1-\gamma) \quad (2)$$

or Coles' wake law [7]

$$\frac{u}{v_*} = \left(\frac{u}{v_*}\right)_{in} + \frac{n}{k}W. \quad (3)$$

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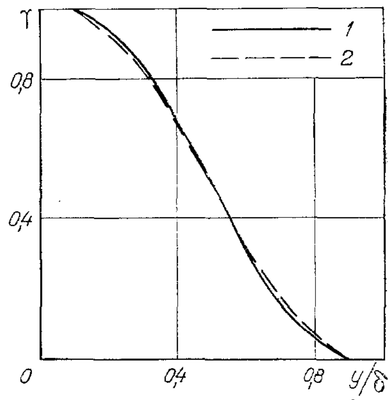

Fig. 1. Thompson function γ: 1) according to Galbraight and Head; 2) according to Eq. (8).

Let us first look at the determination of shear stress by means of Eq. (2).

As was shown in [6, 8] and other studies, the two-parameter Thompson profile approximates well experimental data on velocity distribution in the boundary layer. We also found evidence of this in our analysis of the study results presented at the Stanford conference and the investigations [9, 10]. Comparison of theoretical velocity profiles with experimental profiles for 125 experiments — including 25 different equilibrium, nonequilibrium, and relaxation flows — within a broad range of pressure gradients showed a nearly complete agreement.

According to Galbraight and Head [8], the "weight" function γ is equal to unity within the range $0 < y/\delta \leqslant 0.05$ and equal to zero close to the outer edge of the boundary layer at $0.95 < y/\delta \leqslant 1$. The interval $0.05 < y/\delta \leqslant 0.95$ is divided into three sections. The function γ is described in each of these sections by a second-order polynomial. The coefficients of the polynomial are chosen so that continuity of the first derivative is assured at the conjugate points.

To simplify the calculations, we will introduce a function which approximates γ over the entire interval (0.05)-(0.95).

Let us rewrite Eq. (3) in the form

$$\frac{u - u_{\text{in}}}{v_*} = \frac{n}{k} W. \tag{4}$$

On the outer edge of the boundary layer, where W = 2, we will have

$$\frac{u_1 - u_{\text{in}}}{v^*} = 2 \frac{n}{k}. \tag{5}$$

After dividing (4) by (5) and transforming, we find

$$\frac{u_1 - u}{u_1 - u_{\text{in}}} = 1 - 0.5W. \tag{6}$$

In making a comparison with (2), we are convinced that the functions γ and W are connected by the simple linear relationship

$$\gamma = 1 - 0.5W. \tag{7}$$

We will take the Heinz approximation for W:

$$W = 1 - \cos \pi \frac{y}{\delta}.$$

Since the Thompson function is approximated by the Galbraight and Head polynomials within the range from $y/\delta = 0.05$ to $y/\delta = 0.95$, then we approximate W within the same limits. Then

$$\gamma = 0.5 \left[1 + \cos \frac{10}{9} \pi \left(\frac{y}{\delta} - 0.05 \right) \right]. \tag{8}$$

The functions γ calculated by means of the polynomials and from Eq. (8) nearly coincide (Fig. 1).

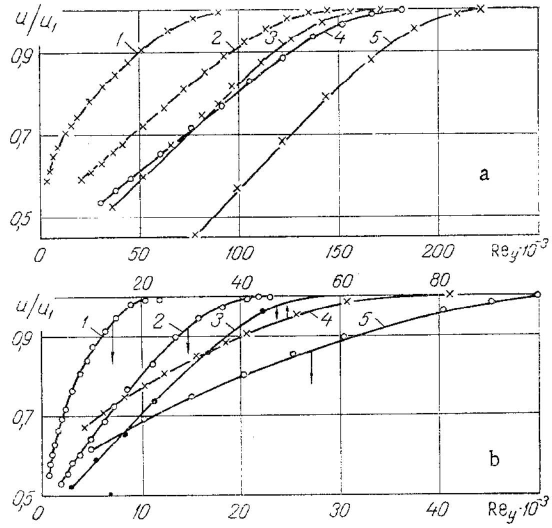

Fig. 2. Calculated (solid lines) and measured (points) velocity profiles: a) 1 — Bradshaw's experiments, x = 0.762 m; 2 — same, x = 1.879 m; 3 — Schubauer and Spengenberger, x = 4.57 m; 4 — Ludwieg and Tillman, x = 3.33 m; 5 — Schubauer and Spengenberger, x = 5.08 m; b) 1 — our experiments, x = 0.3 m; 2 — same, x = 0.7 m; 3 — Arnal's experiments, x = 0.384; 4 — Ludwieg and Tillman, x = 1.28 m; 5 — Arnal — x = 0.478 m.

It follows from (2) and (8) that, beyond the limits of the buffer layer ($y^+ > 30$), the velocity profile in the boundary layer is described by the equation

$$\frac{u}{u_1} = 0{,}5\left[\left(1+\cos\frac{10}{9}\pi\eta\right)\left(\frac{\omega}{k}\ln \mathrm{Re}_y\omega + B\omega\right) + \left(1-\cos\frac{10}{9}\pi\eta\right)\right], \tag{9}$$

where

$$\eta = \frac{y}{\delta} - 0{,}05,\ \mathrm{Re}_y = \frac{u_1 y}{\nu}.$$

We take the following for the velocity distribution in the laminar sublayer and buffer layer [8]:

$$\text{at} \quad 0 < y^+ < 4 \quad u^+ = y^+, \tag{10}$$

$$\text{at} \quad 4 < y^+ < 30 \quad u^+ = 4.187 - 5.745\ln y^+ + 5.11(\ln y^+)^2 - 0.767(\ln y^+)^3. \tag{11}$$

Equation (11), proposed by Dvorok and modified somewhat in the work [8], ensures continuity of the first derivative at the layer boundaries.

Substituting (9), (10), and (11) into the expression for the displacement thickness and integrating within the corresponding limits on the assumption of a three-layer scheme for the boundary layer, we find

$$\delta^* = \delta\left[0.5 - \omega\left(0.80095 + 1.1943\ln\frac{\delta u_1}{\nu}\omega\right)\right] + 50.7\frac{\nu}{u_1}. \tag{12}$$

Equations (9) and (12) are the basis of the proposed method for determining the coefficient of friction on the wall.

We first use the velocity profile measurements to calculate the displacement thickness δ^*. Directing our attention to the experimental velocity determination, we assign the boundary-layer thickness δ. Knowing δ^*, we find ω from Eq. (12). We then use the known values of ω and $\mathrm{Re}_\delta = u_1\delta/\nu$ and the series of values taken for y/δ to first calculate $\mathrm{Re}_y = \mathrm{Re}_\delta y/\delta$. Then we calculate u/u_1 by means of Eqs. (9). We compare these values with the experimental results. If there is no agreement the calculations are repeated after assigning a new value for δ. It is convenient to calculate the relative velocities u/u_1 at the same values of y/δ that were used in the experiment.

TABLE 1. Comparison of Methods of Determining the Friction Coefficient

Serial No.	Experiments	x, м	H	δ^*, mm	β	$c_{f_C}\cdot 10^5$	$c_{f_{LT}}\times 10^5$	$c_{f_T}\cdot 10^5$	$c_{f_m} 10^5$	$c_f\cdot 10^5$
1	Bradshaw, flow C	0,762	1,39	5,441	1,331	245	238	239	225	238
2	Ludwieg and Tillman, flow 1200	1,282	1,400	5,446	0,582	249	248	249	245	249
3	Authors, $T_u = 2{,}6\%$	0,300	1,469	2,414	0,717	369	370	379	—	$\frac{366}{363}$
4	Arnal, flow A2 $T_u = 5{,}0\%$	0,478	1,490	3,747	1,600	213	228	219	—	$\frac{238}{228}$
5	Bradshaw, flow C	1,829	1,522	15,18	2,678	161	160	158	163	162
6	Authors $T_u = 2{,}4\%$	0,700	1,565	7,464	2,133	232	252	247	—	245
7	Ludwieg and Tillman, fow 1200	3,332	1,648	29,81	6,312	120	121	120	123	122
8	Arnal, flow A1	0,384	1,670	2,306	1,369	205	199	208	—	205
9	Schubauer and Spengenberger, flow 4800	4,572	1,752	46,41	17,615	94	105	100	—	$\frac{106}{97}$
10	Schubauer and Spengenberger, flow 4800	5,080	2,342	97,40	74,38	27	38	35	—	37

To check the validity of the above method of determining wall shear stress from a measured velocity profile in the outer part of a boundary layer, we analyzed experimental data taken from materials from the Stanford conference [11], Arnal's experiments [12] on boundary-layer flow at a high degree of outer-flow turbulence, and our own velocity-distribution measurements.

The experiments used for the analysis embrace a fairly broad range of values of the form factor H, displacement thickness δ^*, and the Clauser parameter $\beta = \delta^*/\tau_W \cdot dP/dx$. The results of the calculations are shown in Table 1 and Fig. 2.

The resulting values of friction coefficient C_f (the last column of Table 1) are compared with the friction coefficient values determined by Clauser's method (D_{f_C}), from Ludwieg's and Tillman's formula ($C_{f_{LT}}$), by means of Thompson curves (C_{f_T}), and by direct measurement (C_{f_m}). In all of the cases of boundary-layer flows examined, complete agreement was obtained among the values of C_f determined by the different methods. The maximum and minimum values of the friction coefficient in the above-examined experiments differ by one order.

The velocity distribution in the outer part of the boundary layer can be approximated by the three-parameter Coles profile. Accordingly, determination of the wall shear stress requires the calculation of three equations. We tried out two variants of systems of equations: the first included Eq. (3), as well as the equations

$$\frac{k\delta^*}{\omega\delta} = 1 + \Pi. \tag{13}$$

$$\frac{k^2}{2\omega^2}\left(\frac{\delta^* - \theta}{\delta}\right) = 1 + 1.590\Pi + 0.75\Pi^2. \tag{14}$$

They were obtained by Coles [7] on the assumption that flow in the laminar sublayer and transitional region deviates negligibly from the wall logarithmic law.

Excluding the parameter Π from (3) and (14) by means of Eq. (13), we obtain two equations which are solved simultaneously, as examined above.

We use Eq. (3) with $y = \delta$ in the second variant. The initial data required in the first variant are the velocity profile and the two integral boundary-layer characteristics δ^* and θ, while the initial data in the second variant are the velocity profile and δ^*. The two variants gave roughly the same values for the friction coefficient. The calculations were performed for three flows. The results are shown on lines 3, 4, and 9 in Table 1. The values of C_f in the last column are given in fractional form for these lines: the numerator gives the results calculated using the Thompson profile, while the denominator shows the results obtained using the wake law of Coles (second variant).

It can be seen from Table 1 that the two methods give generally the same results. We did not conduct a detailed comparative analysis of the methods. It can only be suggested that the method based on the Coles wake law is less accurate due to the assumption made in calculating the integral characteristics δ^* and θ.

NOTATION

C_f, friction coefficient; u, current velocity over the thickness of the boundary layer; u_1, velocity at the outer boundary of the boundary layer; τ_W, wall shear stress; $v_* = \sqrt{\tau_W/\rho}$, dynamic velocity; $u^* = u/v_*$; $y^* = yv_*/\nu$; Π, parameter in Coles' wake law; $(u/u_1)_{in}$, $(u/v_*)_{in}$, velocity distribution in the wall region [Eqs. (10), (11), (1)].

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